On ramp closed orbit correction in SIS18 synchrotron of GSI

Sajjad Hussain Mirza[1,2]

Contributors: Dr. Rahul Singh[1]

Dr. Peter Forck[1]

[1]Beam Instrumentation Group,
GSI Helmholtz Centre for Heavy Ion Research
GSI, Darmstadt

[2]TEMF, TU, Darmstadt
Germany

Outline

1. Introduction to GSI and FAIR synchrotrons
2. Upgradation of SIS18 ring to integrate into FAIR
3. General discussion on closed orbit correction
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    2. DFT based decomposition (a new approach)
4. General requirements for SIS18 (comparison to light sources)
5. Closed orbit perturbations at SIS18
6. Machine model mismatch at SIS18 (correctability and stability)
    1. On-ramp model drift
    2. Intensity dependent tune shift and beta beating
7. System identification
8. Outlook

GSI: brief introduction

Heavy ion research facility,
member of Helmholtz Association (the largest German science organization)

Facilities include:

- Linear accelerator UNILAC
- Heavy Ion synchrotron SIS18
- Experimental Storage ring ESR
- Fragment separator FRS
- High energy laser Phelix
- Several large spectrometers and detector systems
- Medical radiation equipment for cancer treatment (up to 2005)

Six new elements discovered at GSI

Bohrium (107)
Hassium (108)
Meitnerium (109)
Darmstadtium (110)
Roentgenium (111)
Copernicium (112)

Research Areas include:

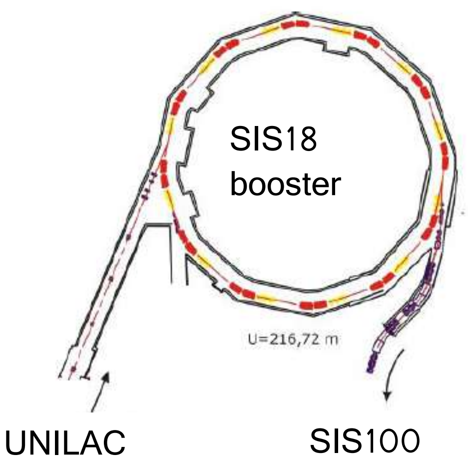
Nuclear Physics
Particle Physics
Plasma Physics
Biophysics and medicine
Material research

FAIR: An extension of GSI

(under construction)

Main purpose:

- High intensity pulsed ion beams
- Secondary beams of rare-isotopes
- Proton beams into antiproton beams

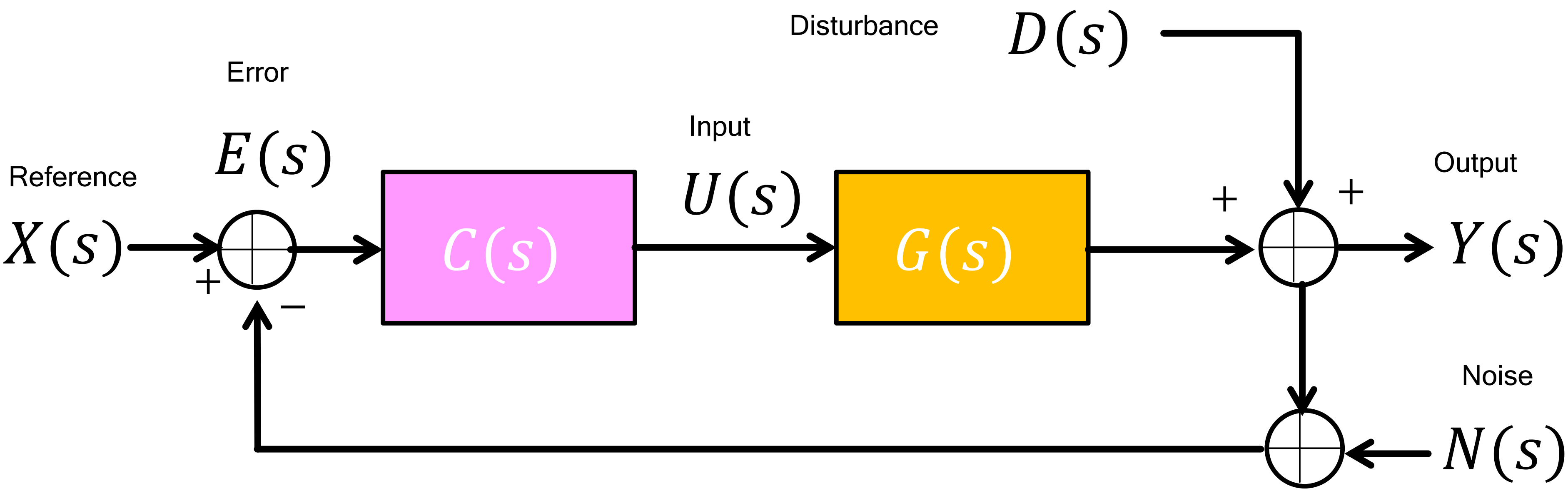


- Lower charge state to avoid space charge effects at high intensities
- Vacuum improvements
- More control on beam quality to deliver more intensity to SIS100 (Closed orbit care)

Synchrotrons: SIS18 and SIS100 (FAIR parameters)

Parameter/Ring	SIS18	SIS100
Circumference (m)	218	1084
Magnetic rigidity (Tm)	18	100
Injection energy	11 MeV/u for U^{28+} (today U^{73+}) 70 MeV/u for protons	200 MeV/u for U^{28+} 4.5 GeV/u for protons
Extraction energy	200 MeV/u for U^{28+} (today ~800 MeV/u) 4.5 GeV/u for protons	2.7 GeV/u for U^{28+} 29 GeV/u for protons
Beam intensity (per pulse)	1.5. 10^{11} ions (today 4×10^{9}) 5. 10^{12} protons	5. 10^{11} ions 4. 10^{13} protons
Magnets	Normal conducting	Super conducting
Ramp rate (max)	10 T/s (variable)	4 T/s
Rep. frequency (Hz)	2.7	0.7
Beam size	5-30 mm (MTI) (1σ)	20-30 mm (1σ)

Closed orbit feedback system

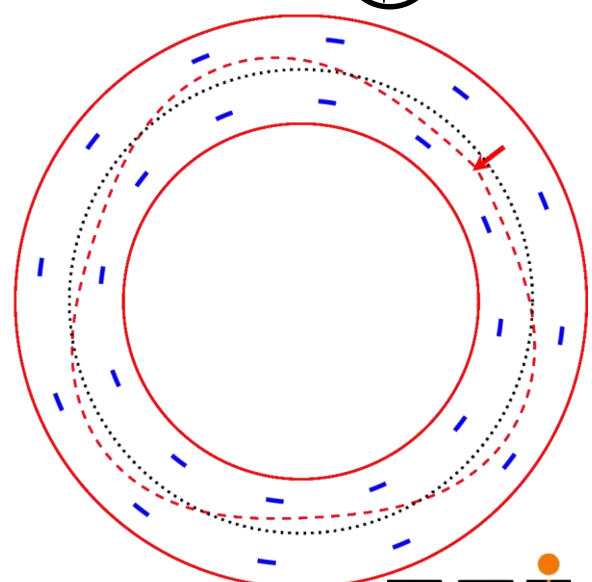


$X(s)$ = Design orbit (zero transverse off-set)

$Y(s)$ = Corrected orbit

$C(s)$ = Controller

$G(s)$ = System model

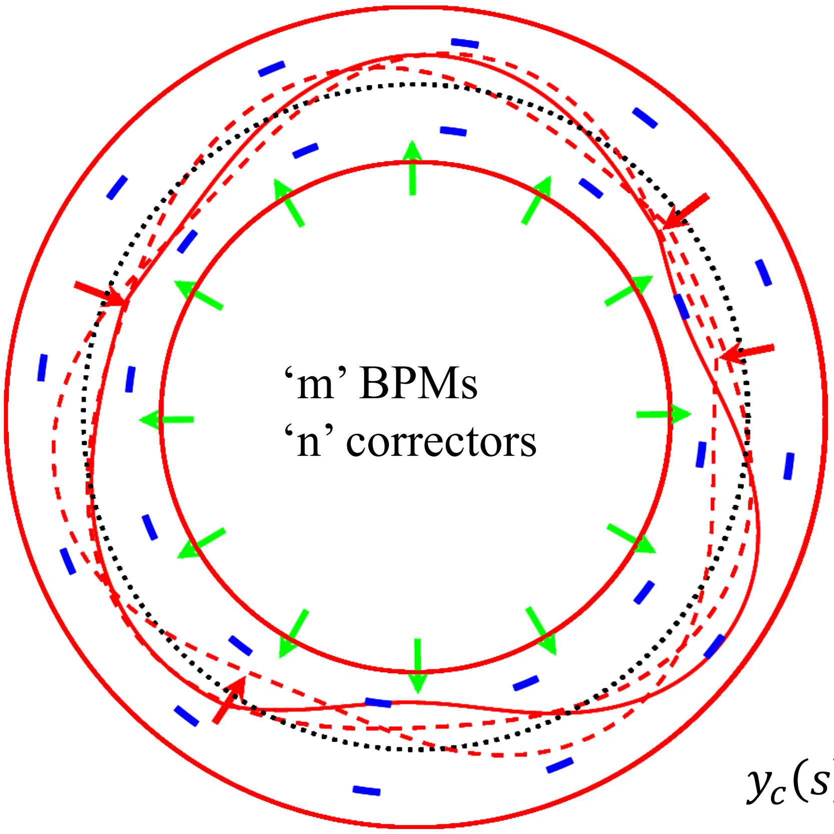


$$G(s) = g(s)\,\boldsymbol{R}$$

$g(s)$ = Temporal response of all hardware in the loop

$\boldsymbol{R}$ = Spatial model of the machine

Closed orbit perturbation (distortion)



Single error perturbed orbit is

$$y_c(s) = \theta \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{s0}| - \pi Q_y)$$

θ is the kick provided by field error
$\beta(s)$ is the beta function at kick location
$\mu(s)$ is the phase advance
Q is the tune of the machine

$$y_c(s) = \sum_{i=1}^{N} \theta_i \frac{\sqrt{\beta(s_i)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{si}| - \pi Q y)$$

R is called the orbit response matrix

$$[\mathbf{Y}]_{m\times 1} = [\boldsymbol{R}]_{m\times n}[\Theta]_{n\times 1}$$

Orbit response matrix (ORM) based correction

Matrix containing proportionality constants can be calculated or measured separately

$$[\mathbf{Y}]_{\boldsymbol{m}\times\mathbf{1}} = [\boldsymbol{R}]_{\boldsymbol{m}\times\boldsymbol{n}}[\Theta]_{\boldsymbol{n}\times\mathbf{1}}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ \cdot \\ Y_{m-1} \\ Y_m \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots & R_{1n} \\ R_{21} & R_{22} & R_{23} & \cdots & R_{2n} \\ R_{31} & R_{32} & R_{33} & \cdots & R_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{m-1,1} & R_{m-1,2} & R_{m-1,3} & \cdots & R_{m-1,n} \\ R_{m1} & R_{m2} & R_{m3} & \cdots & R_{mn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdot \\ \cdot \\ \cdot \\ \theta_{n-1} \\ \theta_n \end{bmatrix}$$

Then apply the negatives of the calculated corrector strengths

$$\begin{bmatrix} -\theta_1 \\ -\theta_2 \\ -\theta_3 \\ \cdot \\ \cdot \\ \cdot \\ -\theta_{n-1} \\ -\theta_n \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdot \\ \cdot \\ \cdot \\ \theta_{n-1} \\ \theta_n \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots & R_{1n} \\ R_{21} & R_{22} & R_{23} & \cdots & R_{2n} \\ R_{31} & R_{32} & R_{33} & \cdots & R_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{m-1,1} & R_{m-1,2} & R_{m-1,3} & \cdots & R_{m-1,n} \\ R_{m1} & R_{m2} & R_{m3} & \cdots & R_{mn} \end{bmatrix}^{-1} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ \cdot \\ Y_{m-1} \\ Y_m \end{bmatrix}$$

1. ORM is not always invertible (for example rectangular)
2. Calculated corrector values are beyond the corrector magnet range

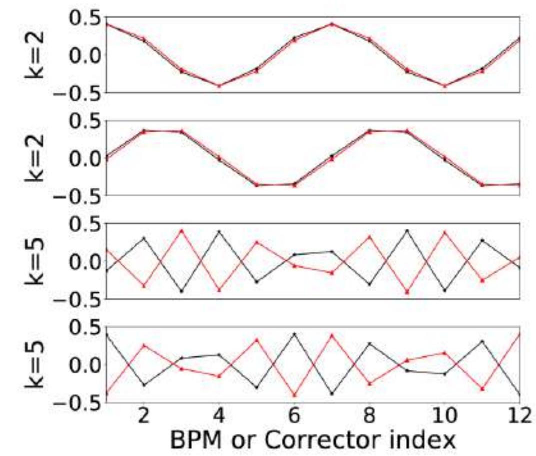
SVD for ~ ill conditioned ORMs

Y. Chung, "Closed orbit correction using singular value decomposition of the response matrix", (Argonne National Laboratory, IL, 1993)

Singular Value Decomposition (SVD)

$$R = USV^T$$

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}^T$$



s_i are called singular values arranged as $s_1 > s_2 > s_3 \ldots . > s_n$

U and V are orthogonal matrices such that

$$U^{-1} = U^T \text{ and } V^{-1} = V^T$$

where the columns of *U* and *V* are the eigenvectors of RR^T and R^TR

Which helps to find inverse R^{-1} (if *R* is invertible) as

Pseudo-inverse

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix}^{-1} = \begin{bmatrix} V_{11} & \cdots & V_{1m} \\ \vdots & \ddots & \vdots \\ V_{m1} & \cdots & V_{mm} \end{bmatrix} \begin{bmatrix} 1/s_1 & \cdots & 0 \\ \vdots & 1/s_2 & \vdots \\ 0 & \cdots & 1/s_n \end{bmatrix} \begin{bmatrix} U_{11} & \cdots & U_{1n} \\ \vdots & \ddots & \vdots \\ U_{n1} & \cdots & U_{nn} \end{bmatrix}^T$$

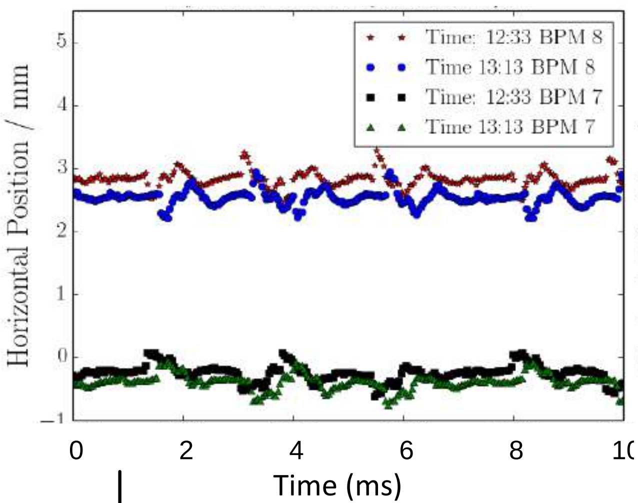
William H. Press, Numerical recipes; The art of scientific computing (2007) Cambridge university press

Comparison to the state-of-the art realizations
(Light sources)

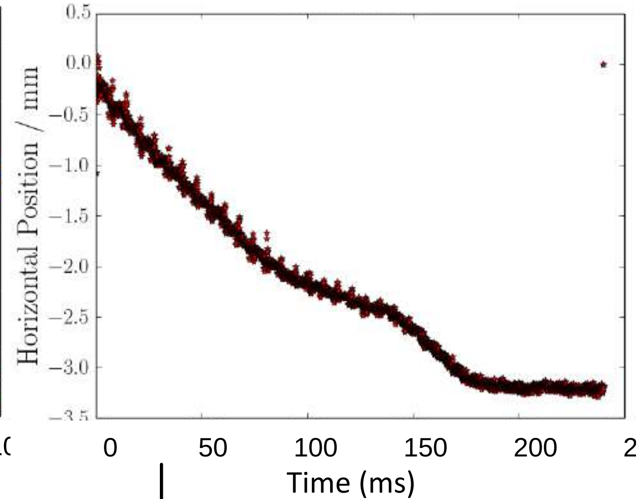
Parameter	State-of-the art (light sources)	SIS18
Stability criteria (vertical plane)	Less than 1 μm (10% of beam size ~ 10 μm)	Less than 1 mm (10% of beam size ~10 mm)
Bandwidth	~ 1 – 250 Hz	>600 Hz - 1 kHz
Sources	Mechanical vibrations /power supply ripples	Higher harmonics of PS ripples/ hysteresis
On-ramp correction	Not needed(?)	Primary plan
Lattice settings	Fixed	Systematic variation
Flexibility of operations	Electron beams Fixed energies Almost fixed intensities	Protons to heavy ions Variable beam intensities Variable beam energies
BPM failure/ malfunction	Less probability(?)	More probability due to high radiation
Beta beating	Lattice model more understood	Variable optics

Closed orbit perturbations in SIS18

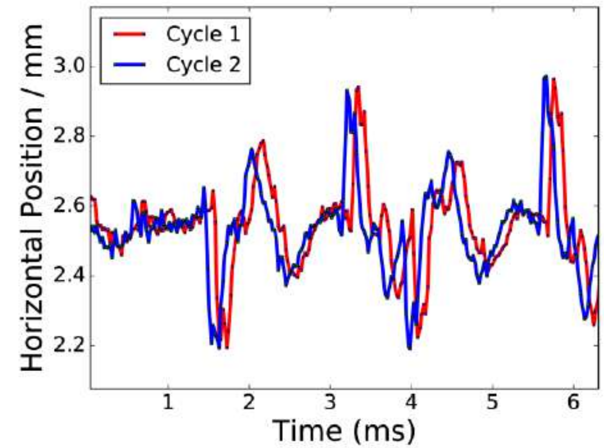
At injection



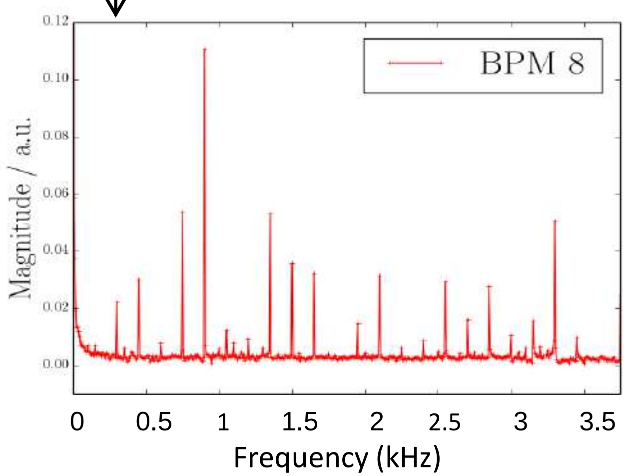
During ramp



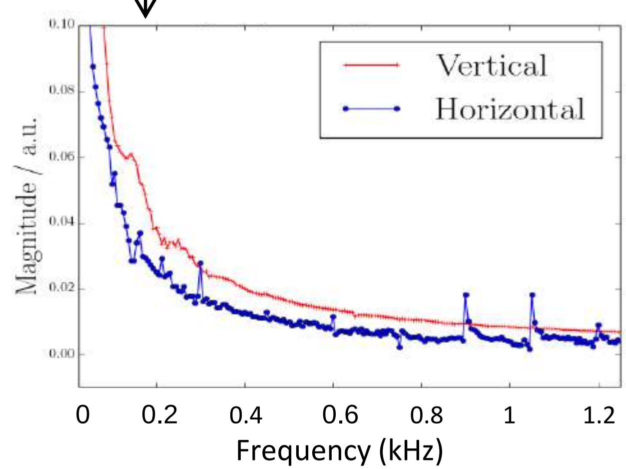
Ramps of two cycles



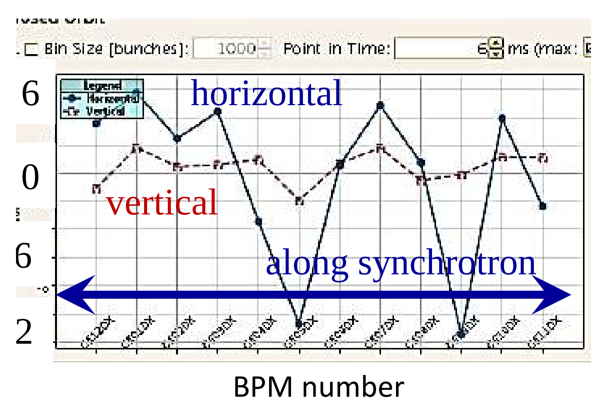
Fourier Transform



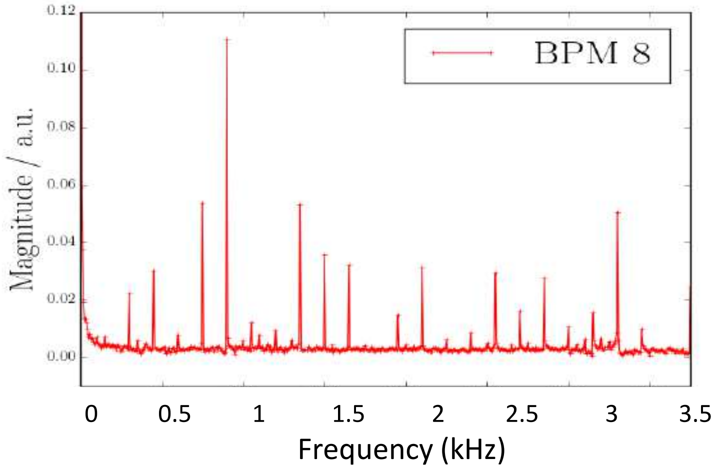
Fourier Transform



Static perturbation



High Frequency ripples and bandwidth requirements



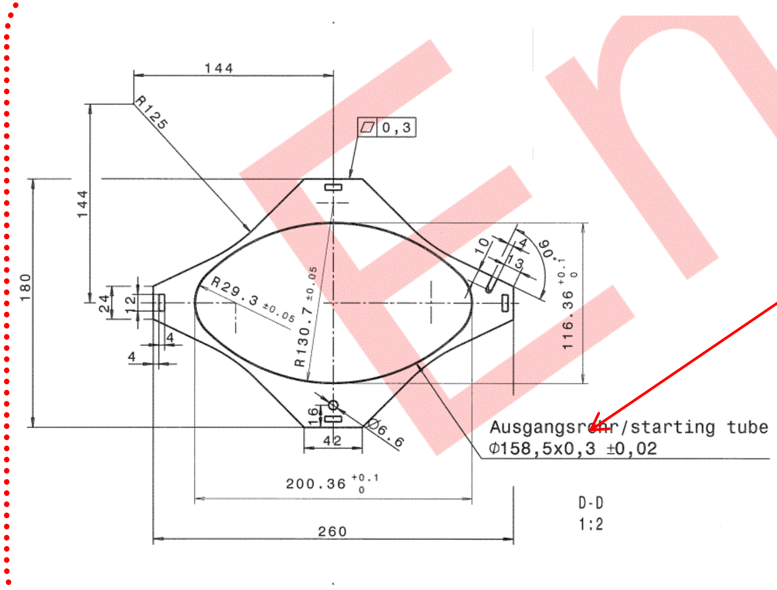
To avoid eddy currents during fast ramps
Wall thickness of
Quadrupole vacuum chamber = 300 μm
Dipole vacuum chamber = 400 μm

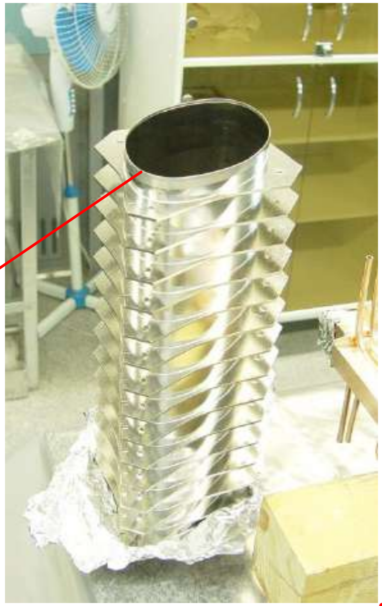
$$G(s) = \frac{1}{1 + \tau s} \qquad \tau = 1.03\,\frac{\mu_0 \sigma a d}{2}$$ for elliptical chamber

For quadrupole chamber

d= 0.3 mm
a= 100.18 mm
τ= 25.51 μs

f_c = 6.23 kHz





- Very high frequency power supply ripples can be coupled to the beam.
- For on ramp correction the reaction time is below 1 ms requiring correction up to 1 kHz.

S. H. Kim. Calculation of pulsed kicker magnetic eld attenuation inside beam chambers. Technical Report LS-291, Advanced Proton Source, 2001.

SIS18 synchrotron's "machine model"

- **SIS18 ring:** a strictly periodic lattice > 12 identical sections
- **Each section:** Two dipoles and a set of three quadrupole

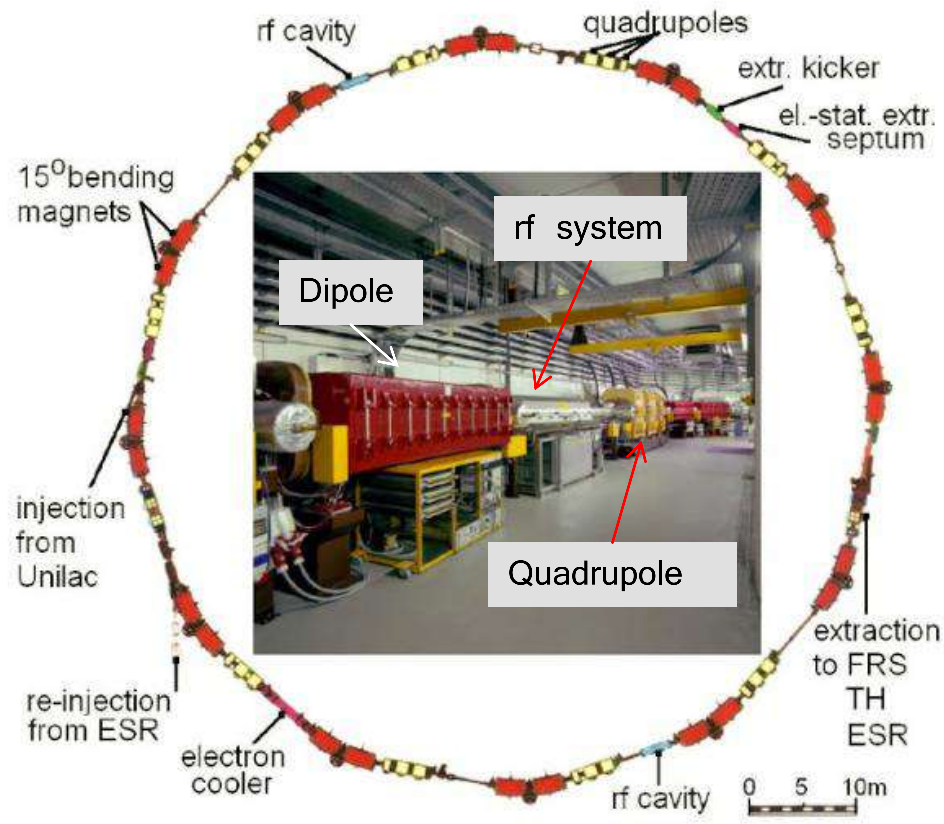
$$\beta_{bpm1} = \beta_{bpm2} = \beta_{bpm3} \dots \dots = \beta_{bpm12}$$

$$\beta_{corr1} = \beta_{corr2} = \beta_{corr3} \dots \dots = \beta_{corr12}$$

$$\Delta\mu_{bpm} = constant \qquad \Delta\mu_{corr} = constant$$

$$R = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & \cdots & R_n \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$

Each row is cyclic shift of previous row and all diagonal elements are identical.



Such a square matrix is called **Circulant Matrix**

Exploiting the Circulant symmetry of SIS18

$$R = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & \cdots & R_n \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$

Inverse is straightforward

$$R^{-1} = F^* H^{-1} F$$

$$H^{-1} = \text{diag}\left(\frac{1}{\sigma_k}\right),\ k=1\ldots n$$

$$\sigma_k = \sigma_{rk} + j\,\sigma_{ik} = \sum_{n}^{N-1} R_n\, e^{-j2\pi kn/N}$$

$$R = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$

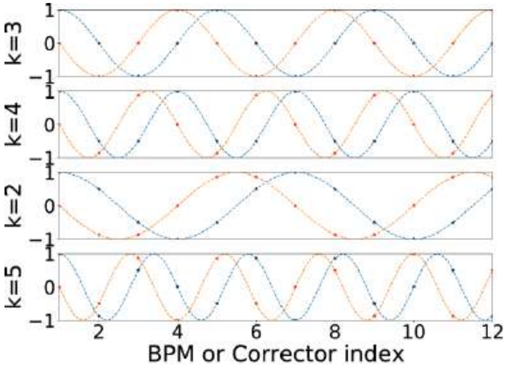
Standard Fourier matrix containing DFT modes

$$F_k = F_{kc} + jF_{ks} \qquad F_{ks} = \sin\left(\frac{2\pi km}{n} + \varphi_k\right)$$

Equivalence of SVD and DFT

DFT:

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$



$$\varphi_{dk} = \text{phase}(\sigma_k)$$

$$s_k = |\sigma_k| = \sqrt{{\sigma_{rk}}^2 + {\sigma_{ik}}^2}$$

SVD:

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}$$

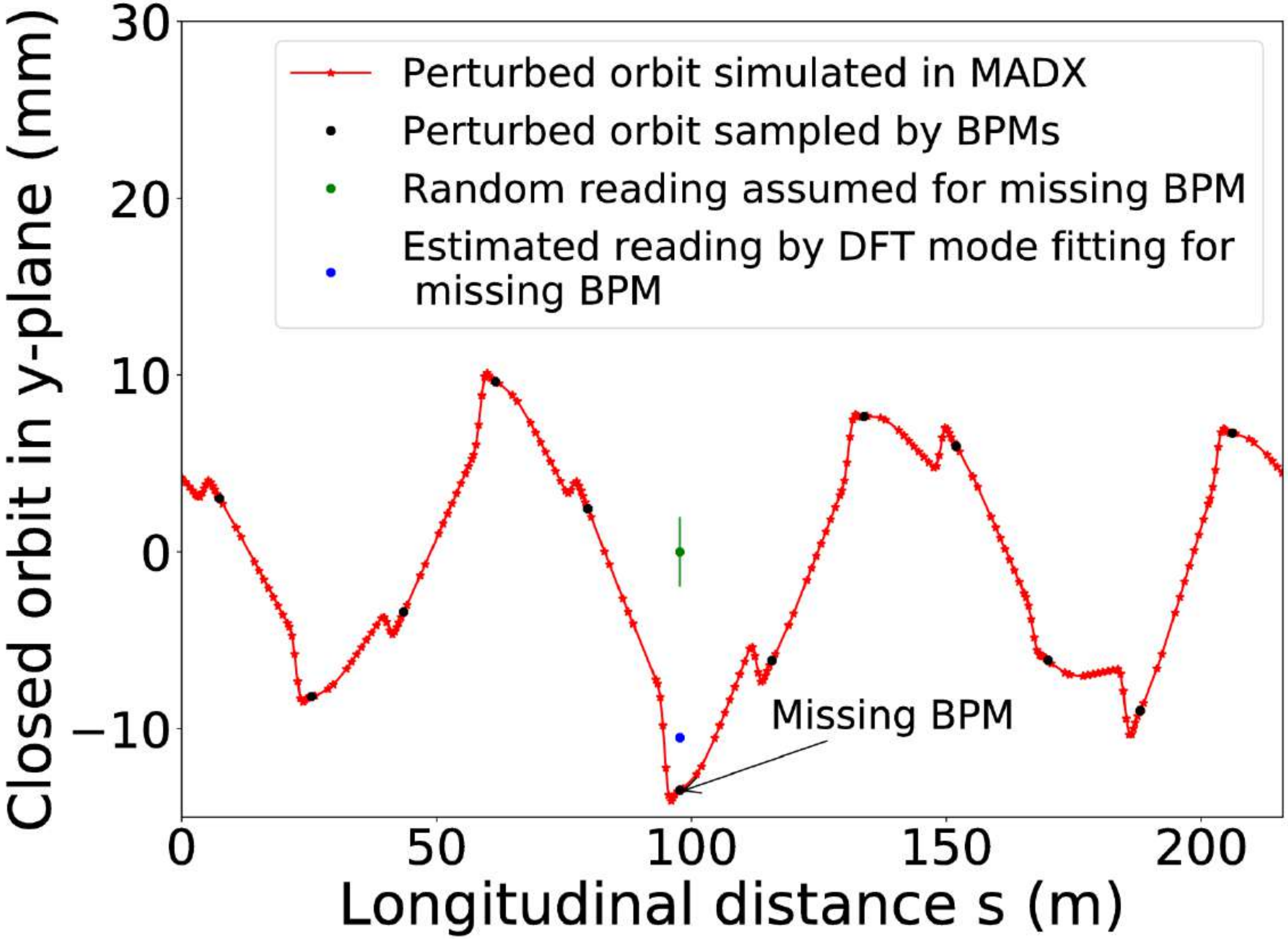
Why to do SVD when Circulant symmetry exits?

One quick application: Missing BPM scenario

$$F_{ks} = \sin\left(\frac{2\pi km}{n} + \varphi_k\right)$$

$$F_{kc} = \cos\left(\frac{2\pi km}{n} + \varphi_k\right)$$

Fit the measured orbit at functioning BPMs and fit over dominant Fourier modes



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One quick application: Missing BPM scenario

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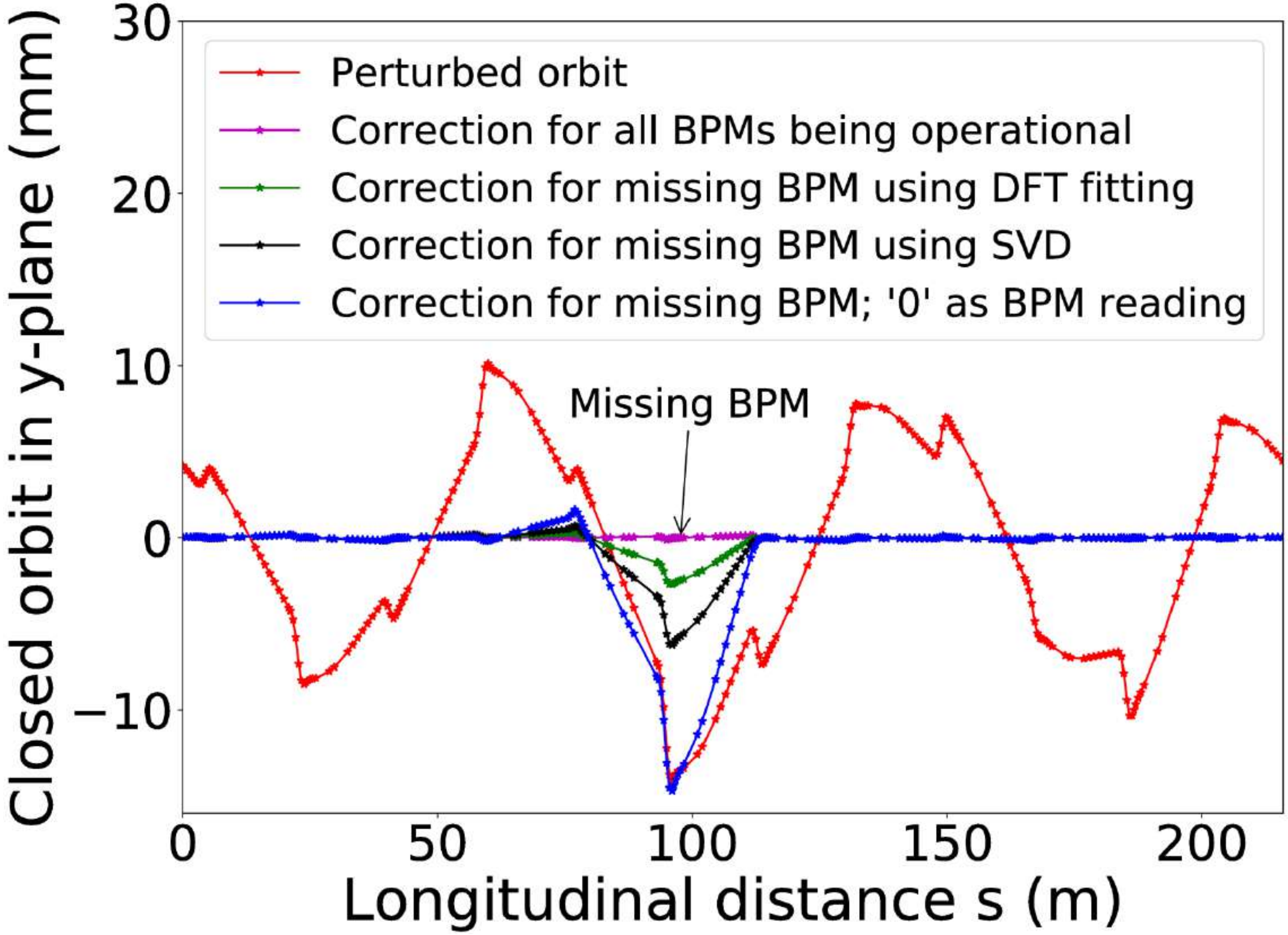
Fit the measured orbit at functioning BPMs and fit over dominant Fourier modes



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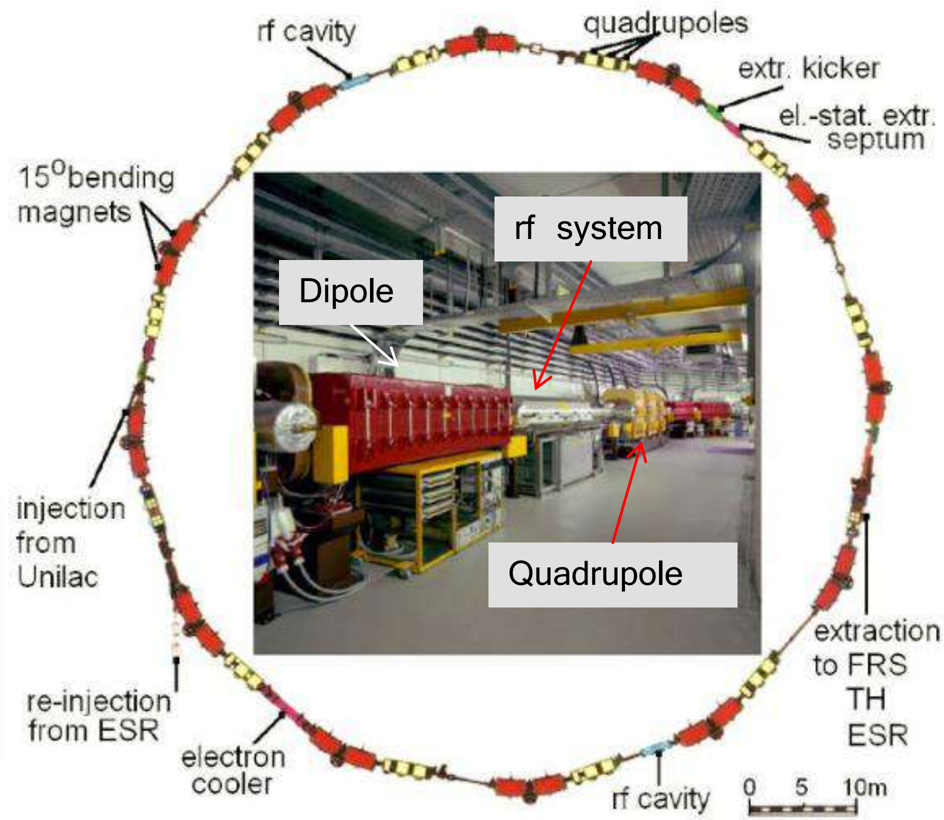
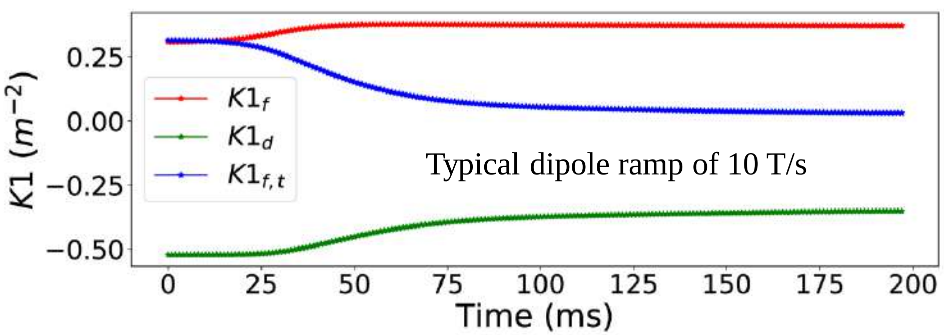
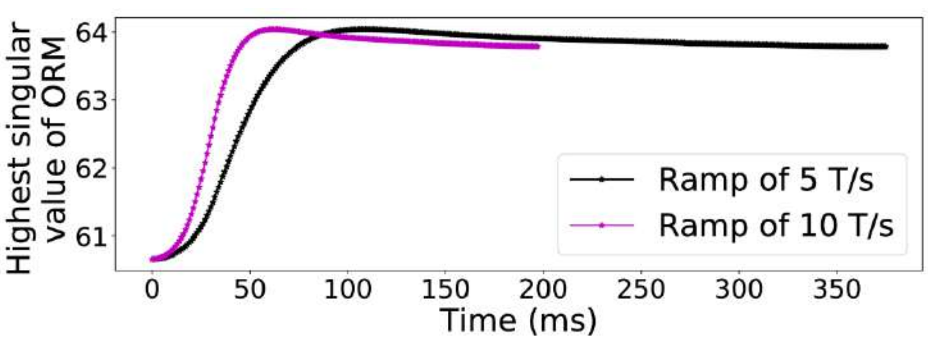
SIS18 machine model changes and uncertainties

- **SIS18 ring:** a strictly periodic lattice > 12 identical sections
- **Each section:** Two dipoles and a set of three quadrupole
- Triplet quadrupole configuration > injection > high acceptance > multi−turn injection







- The triplet to doublet transition is made to keep the tune constant.

SIS18 machine model changes and uncertainties

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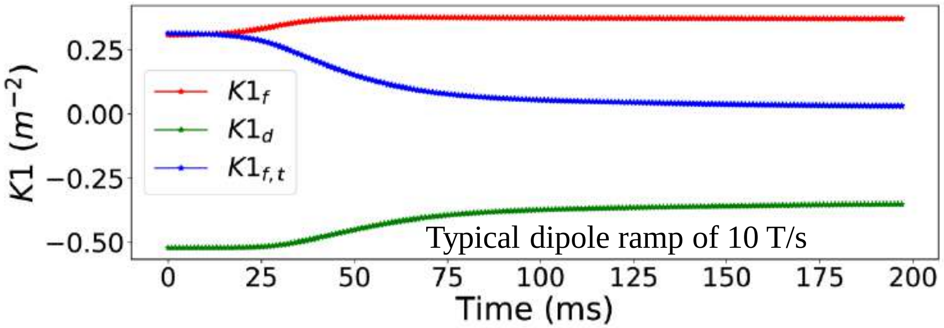
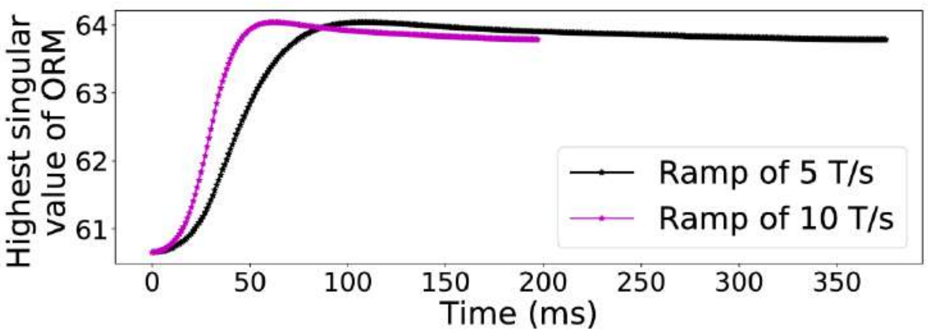
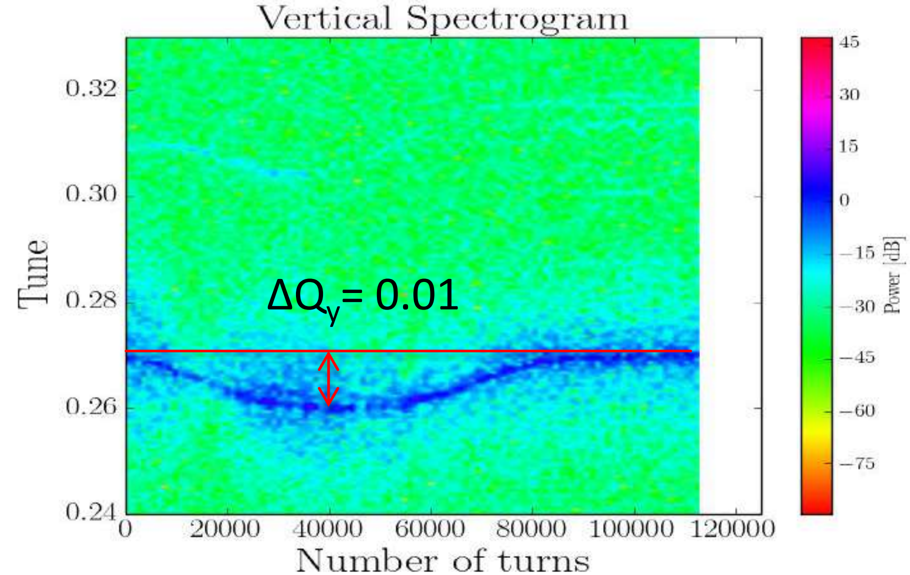
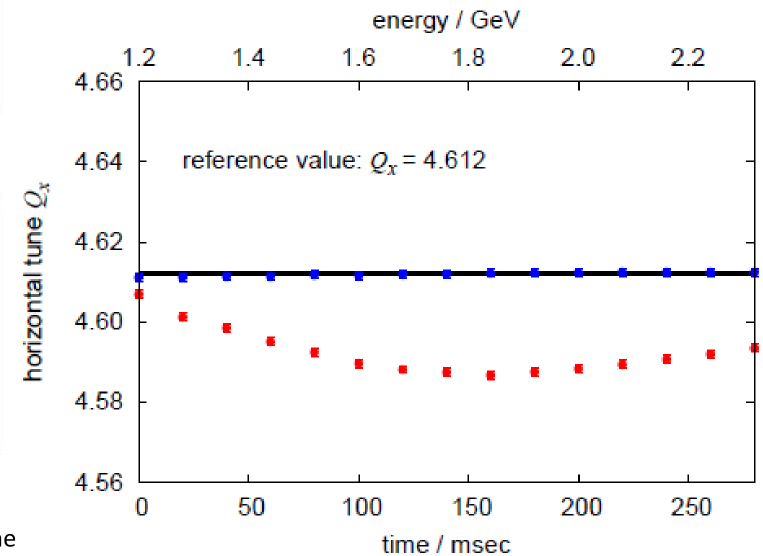








M. Eberhardt et al., "Measurment and correction of the longitudinal and transverse tune during the fast energy ramp at ELSA“ in *Proc. IPAC’10*, Kyoto, Japan, 2010, paper MOPD085.

Effect of model mismatch on the orbit correction

$$[\mathbf{R}]_{m\times n} = \frac{\sqrt{\beta_m \beta_n}}{2\sin(\pi Q)} \cos(\pi Q - |\mu_m - \mu_n|)$$

If **R** = actual machine model
R' = assumed model used to calculate the corrector strengths θ
Δy_0 = initial perturbed orbit then
r_1 = the residual orbit after one iteration

$$r_1 = \Delta y_0 - \mathbf{R}\theta$$

$$r_1 = \Delta y_0 - \mathbf{R}\mathbf{R}'^{-1}\Delta y_0$$

$$r_1 = \left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right)\Delta y_0$$

r_1 ∝ the model mismatch
∝ *n* (number of iterations required for convergence)
gives a hint of the correctability and stability criteria after n iterations

The residual after *n* iterations becomes

$$r_n = \left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right)^n \Delta y_0$$

If any Eigenvalue of $\left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right) > 1$ closed correction after n iteration will lead to instability.

Effect of model mismatch on the orbit correction
(On-ramp model drift)

$$[\mathbf{R}]_{m\times n} = \frac{\sqrt{\beta_m\beta_n}}{2\sin(\pi Q)}\cos(\pi Q - |\mu_m - \mu_n|)$$

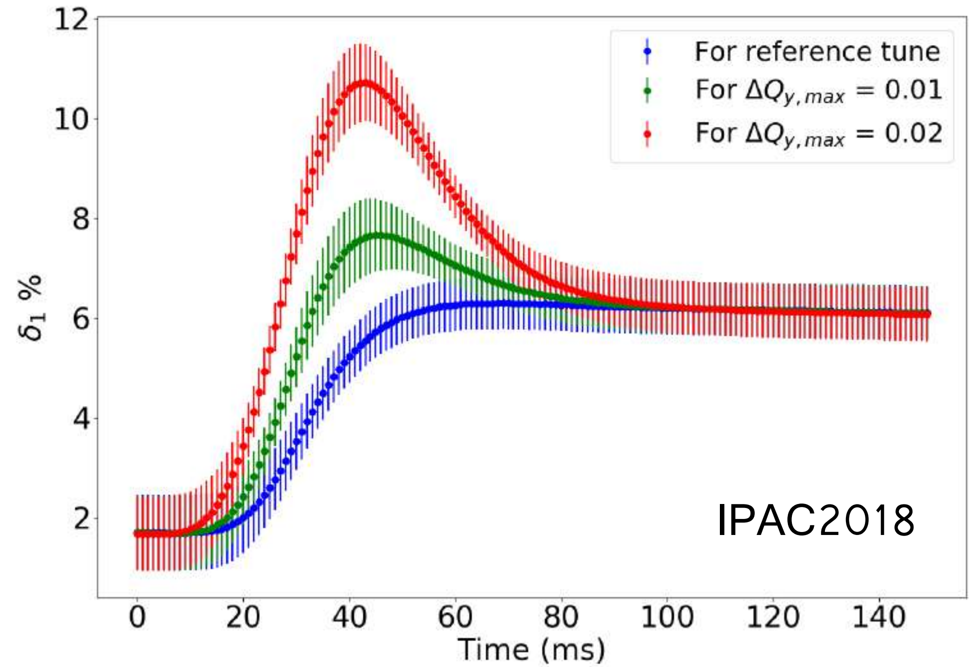
$$r_1 = \Delta y_0 - \mathbf{R}\theta$$

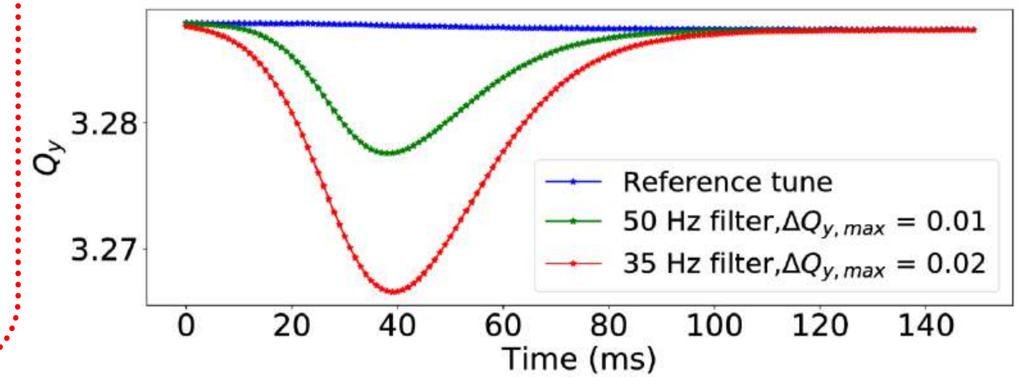
$$r_1 = \Delta y_0 - \mathbf{R}\mathbf{R}'^{-1}\Delta y_0$$

$$r_1 = \left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right)\Delta y_0$$

$$\delta_1 = \frac{r_{1,RMS}}{\Delta y_{0,RMS}}$$

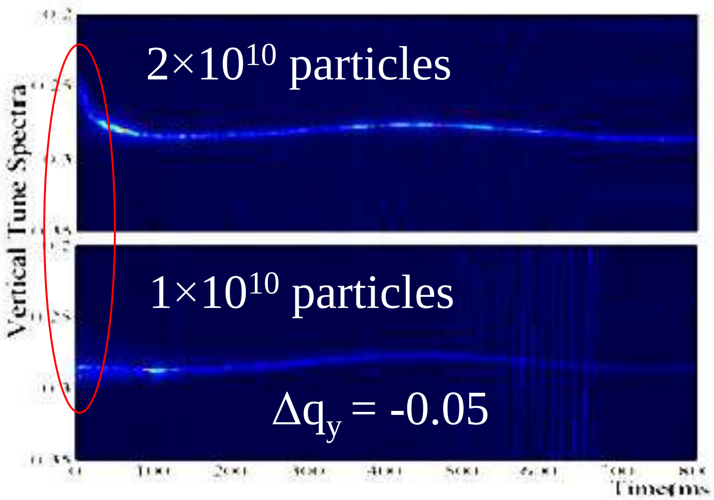
- **R** corresponding to injection settings was used for orbit correction throughout the ramp.
- 1000 orbit were generated at each time step by Gaussian distribution of random misalignments of quadrupoles.

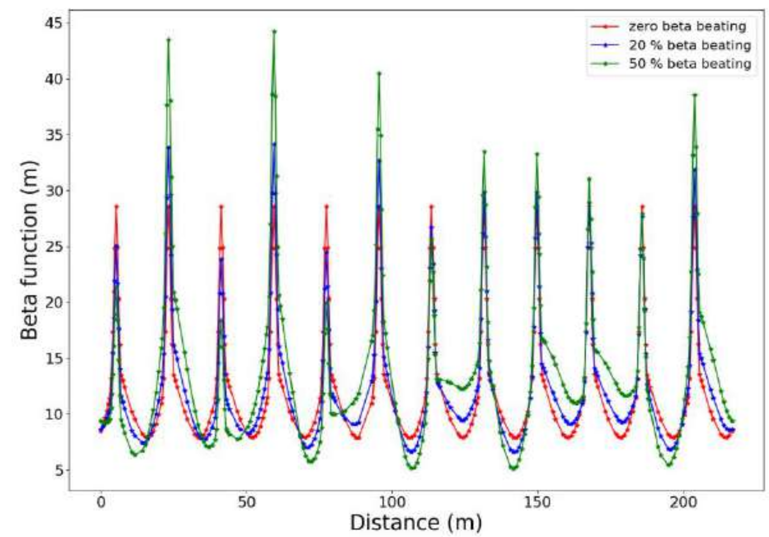




Effect of model mismatch on the orbit correction

(Tune shifts and beta beating)





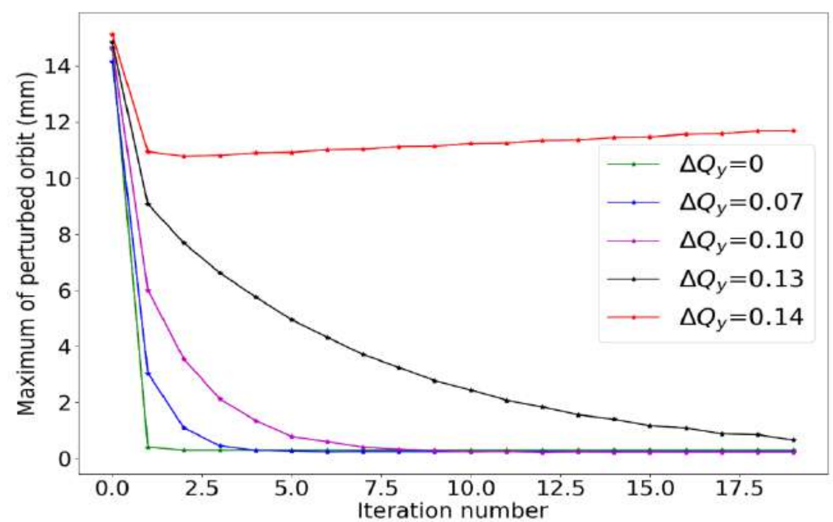
$$r_1 = \Delta y_0 - \mathbf{R}\theta$$

$$r_1 = \Delta y_0 - \mathbf{R}\mathbf{R}'^{-1}\Delta y_0$$

$$r_1 = \left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right)\Delta y_0$$

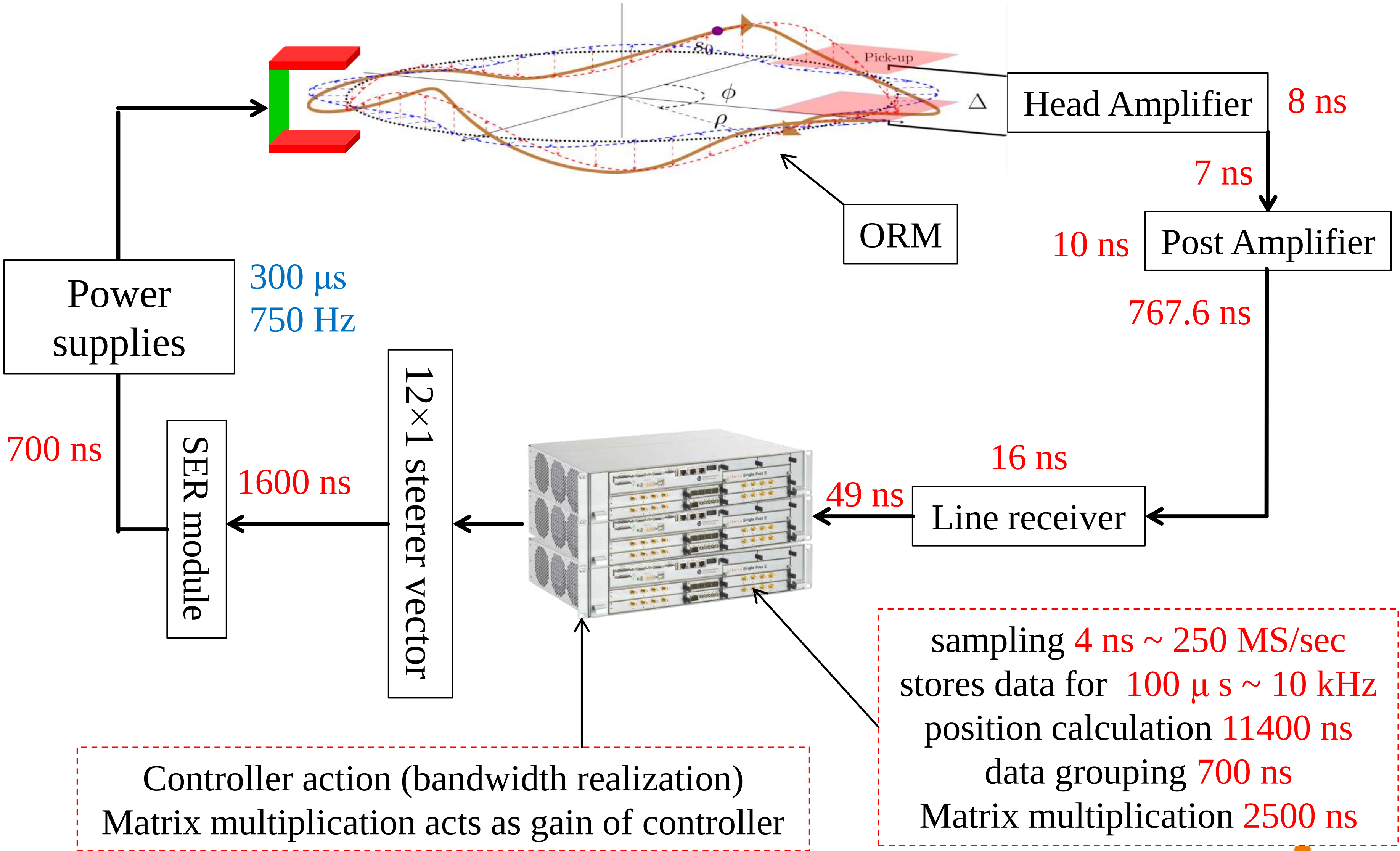
The residual after n iterations becomes

$$r_n = \left(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}\right)^n \Delta y_0$$



R. Singh, "Tune measurement at GSI SIS-18: Methods and Applications" PhD Thesis, TU Darmstadt 2014.

Measurement of transfer functions of hardware
(System identification)



Project layout